Arithmetic Theory of Everything: A Spectral–Topological Framework for Fundamental Constants, Particle Masses and Cosmological Structure

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Abstract

We present the *Arithmetic Theory of Everything* (AToE), a spectral–topological framework in which physical reality is a projection of an underlying arithmetic structure. The eigenvalues of a selfadjoint Dirac operator on a prime Fock space are identified with the imaginary parts of the nontrivial zeros of the Riemann zeta function. Fundamental constants, the lepton and baryon mass hierarchy, and the large–scale structure of the Universe arise as resonances and projection effects of this spectrum into a 12–dimensional dodecahedral geometry. We derive closed expressions for several constants (including the electron mass and mass ratios) and for a set of cosmological observables (CMB anomalies and a specific value of cosmic birefringence), and we discuss concrete experimental tests.

1 Introduction

The standard model of particle physics and ΛCDM cosmology successfully describe a wide range of phenomena, yet treat many dimensionless quantities—such as the fine structure constant α, the electron mass in Planck units, and the cosmological constant—as external inputs. AToE seeks to reduce these "free" parameters to arithmetic necessities by embedding physics into a spectral framework governed by the Riemann zeta function.[1]

We develop a single coherent model that integrates: (i) a prime Fock space carrying a selfadjoint Dirac operator; (ii) a 12–dimensional dodecahedral projection encoding spacetime topology; and (iii) a 137–step resonance structure relating these spaces to the observed values of fundamental constants.

1.1 Axioms of the Arithmetic Theory of Everything

AToE rests on the following explicit axioms.

Axiom 1 (Prime Fock space). Physical states are represented by vectors in the symmetric Fock space $\mathcal{H}_F$ over the one–particle space $\mathcal{H}_1 = \ell^2(P)$, where P is the set of prime numbers.

Axiom 2 (Spectral Riemann Axiom). There exists a selfadjoint Dirac operator D on $\mathcal{H}_F \oplus \mathcal{H}_F$ whose nonzero eigenvalues occur in pairs $\pm\gamma_n$, where γ_n are the imaginary parts of the nontrivial zeros $\frac{1}{2} + i\gamma_n$ of the Riemann zeta function.

[1]For background on zeta zeros and random matrix statistics see e.g. Montgomery and Odlyzko; for dodecahedral cosmology see Luminet *et al.*.

Axiom 3 (Arithmetic phase). Interactions between prime modes $p, q \in P$ are encoded in a nonabelian phase

$$\Phi(p,q) = \exp\left(2\pi i \, \frac{pq}{\log(pq)}\right),$$

and the statistical correlations of the eigenvalues γ_n follow the GUE universality class.

Axiom 4 (12–dimensional dodecahedral projection). Observable spacetime arises from a projection $P : \mathcal{A}_{137} \to \mathbb{R}^{12}$ of a 137–dimensional arithmetic state space $\mathcal{A}_{137}$ onto a 12–dimensional manifold with dodecahedral symmetry and 36° sector quantization.

Axiom 5 (Master–matrix construction of constants). Each fundamental constant P is given by

$$P = K(\gamma_n)\, \Phi_{\text{Top}}\, \eta_A\, N_{\text{SI}},$$

where $K(\gamma_n)$ is an arithmetic kernel built from the zeta zeros, Φ_{Top} encodes the dodecahedral topology, η_A is an arithmetic viscosity, and N_{SI} converts to SI units.

Axiom 6 (Dynamic interface dimension). A twelfth effective dimension D_{12} acts as an information interface with characteristic frequency

$$f_{D12} = \left(\frac{\gamma_{12}}{\gamma_1\, 100\pi}\right)^{-1} \approx 8.0 \text{ Hz},$$

governing coupling between the arithmetic background and physical observers.

All other statements in this paper are intended as *derivations* from these axioms plus standard mathematical and physical results.

1.2 Overview of the AToE framework

At a conceptual level, AToE is built on three pillars:

1. **Arithmetic substrate:** physical states are configurations in a Fock space built from prime-number modes.
2. **Spectral dynamics:** a Dirac operator D on this Fock space has eigenvalues λ_n that coincide with the imaginary parts γ_n of the nontrivial zeros of the Riemann zeta function.
3. **Topological projection:** observable spacetime arises as a 12–dimensional dodecahedral projection of a higher–dimensional arithmetic structure, with the number 137 playing the role of a principal resonance index.

The rest of the paper makes these statements precise and derives quantitative predictions.

2 Prime Fock space and Dirac operator

2.1 One–particle Hilbert space

Let $P = \{2, 3, 5, 7, 11, \dots\}$ be the set of prime numbers. We define the one–particle Hilbert space as

$$\mathcal{H}_1 = \ell^2(P) = \left\{ \psi : P \to \mathbb{C} \,\middle|\, \sum_{p \in P} |\psi(p)|^2 < \infty \right\}. \tag{1}$$

Each prime p is interpreted as an elementary arithmetic mode.

2.2 Prime Fock space

The full prime Fock space is the symmetric Fock space over $\mathcal{H}_1$:

$$\mathcal{H}_F = \bigoplus_{n=0}^{\infty} \mathrm{Sym}^n(\mathcal{H}_1), \tag{2}$$

with the vacuum sector ($n = 0$), one–prime sector ($n = 1$), and higher sectors representing increasingly complex arithmetic configurations.

2.3 Dirac operator and the Riemann spectrum

We postulate the existence of a selfadjoint Dirac operator

$$D : \mathcal{H}_F \oplus \mathcal{H}_F \to \mathcal{H}_F \oplus \mathcal{H}_F, \tag{3}$$

whose nonzero eigenvalues occur in $\pm$ pairs and satisfy

$$D\Psi_n = \gamma_n \Psi_n, \tag{4}$$

where γ_n are the imaginary parts of the nontrivial zeros $\frac{1}{2} + i\gamma_n$ of the Riemann zeta function. This *Riemann Axiom* is the central physical postulate of AToE: stability of the vacuum corresponds to the conjectured alignment of all nontrivial zeros on the critical line.

3 Arithmetic phase and GUE statistics

3.1 Two–prime phase

The fundamental interaction between prime modes is encoded in a nonabelian phase function

$$\Phi(p, q) = \exp\left(2\pi i \,\frac{pq}{\log(pq)}\right), \qquad p, q \in P. \tag{5}$$

This phase determines constructive and destructive interference of prime configurations in $\mathcal{H}_F$ and is the arithmetic analogue of gauge phases in quantum field theory.

3.2 Level repulsion and GUE

The pair correlation and spacing distribution of the γ_n are known to match those of the Gaussian Unitary Ensemble (GUE). AToE interprets this "quantum chaos" as a direct manifestation of the nonabelian structure of $\Phi(p, q)$. In particular, the level repulsion parameter $\beta \approx 2.01$ sets a minimal spacing in the spectrum and prevents collapse of arithmetic modes.

4 12–dimensional dodecahedral projection

4.1 From 137 dimensions to 12 sectors

We consider a higher–dimensional arithmetic state space $\mathcal{A}_{137}$ whose coordinates encode 137 independent spectral degrees of freedom. Observation suggests a distinguished resonance after 137 discrete projection steps, corresponding to approximately 13.7 cycles of a 12–sector rotation with step angle 36°. Formally, we introduce a projection

$$P : \mathcal{A}_{137} \to \mathbb{R}^{12}, \tag{6}$$

whose image is a 12–dimensional space with dodecahedral symmetry. Each sector corresponds to a 36° slice and is associated with one of 12 effective dimensions $D_1, \ldots, D_{12}$.

Table 1: AToE master matrix: arithmetic construction of fundamental constants.

Dim	Entity	Arithmetic construction (kernel $K(\gamma_n)$)
D_1	Planck constant h	$K(\gamma_1) = \frac{\gamma_1}{2\pi \ln \gamma_1} \, \Phi_{\mathrm{Fix}} \, \eta_h$
D_2	Speed of light c	$K(\gamma_2) = \frac{\gamma_2}{\gamma_1} \frac{\gamma_2}{\pi e} \, \Phi_{\mathrm{Fix}} \, \eta_c$
D_3	Higgs mass m_H	$K(\gamma_{1,2,3}) = f_H(\gamma_1, \gamma_2, \gamma_3)$
D_4	Vacuum permittivity ϵ_0	$K(\gamma_4) = \frac{\gamma_2^4}{4\pi \ln \gamma_4} \, \Phi_{\mathrm{Top}} \, \eta_\epsilon$
D_5	Newton constant G	$K(\gamma_5) = \exp\bigl(-\gamma_5/(\pi \ln \gamma_5)\bigr) \, \eta_G$
D_6	Strong coupling α_s	$K(\gamma_6) = \frac{\ln \gamma_6}{\sqrt{\gamma_6}} \, \Phi_S \, \eta_s$
D_7	Weak scale M_Z	$K(\gamma_7) = \frac{\gamma_7}{\gamma_1} \, m_H \cos\theta_W \, \eta_Z$
D_8	CMB temperature T_{CMB}	$K(\gamma_8) = \gamma_8 \, \Phi_{\mathrm{Fix}} \, \eta_T$
D_9	Vacuum energy density	$K(\gamma_9) = \ln \gamma_9 \, e^{-\gamma_9} \, \Phi_{\mathrm{Fix}} \, \eta_V$
D_{10}	Cosmological constant Λ	$K(\gamma_{10}) = e^{\gamma_{10}} \, \Phi_{\mathrm{Vac}} \, \eta_\Lambda$
D_{11}	Fine structure constant α^{-1}	$K(\gamma_{11}) = \frac{\gamma_{11}\pi}{\ln \gamma_{11}} \left(\frac{\gamma_{11}}{\gamma_1} + \Psi_{D12} \right) \eta_\alpha$

4.2 Topological factor and the golden ratio

The dodecahedral symmetry is encoded in a fixed topological factor Φ_{Fix}, defined as the determinant of a 36° rotation matrix on the dodecahedral lattice:

$$\Phi_{\mathrm{Fix}} \simeq 2.01. \tag{7}$$

Furthermore, the golden ratio $\varphi = (1 + \sqrt{5})/2$ enters the large–scale metric via

$$R_{\mathrm{Cosmos}} \propto \gamma_{12} \, \varphi^2, \tag{8}$$

linking the 12th zeta zero to the effective curvature radius of a Poincaré dodecahedral space.

5 Master matrix for fundamental constants

Each physical quantity P is expressed as the product of four invariant layers:

$$P = K(\gamma_n) \, \Phi_{\mathrm{Top}} \, \eta_A \, N_{\mathrm{SI}}, \tag{9}$$

where:

- $K(\gamma_n)$ is an arithmetic kernel depending on one or several γ_n;
- Φ_{Top} is a topological factor (often Φ_{Fix});
- η_A is an "arithmetic viscosity" accounting for the coarse–graining from discrete primes to continuous fields;
- N_{SI} is a metrological scaling factor that maps natural units to SI units.

Table 1 summarizes the 11 primary dimensions D_1–D_{11} as constructed in the AToE master matrix.

(Here the specific forms of f_H and the viscosities η_A are to be inserted from your master table.)

6 Lepton and baryon masses

In AToE, particle masses arise as resonances of the Dirac spectrum.

6.1 Electron mass

The rest mass of the electron is expressed as

$$m_e = \frac{\hbar\,\gamma_1}{c\,\ell_P}\,\Psi_{\rm proj}, \tag{10}$$

where ℓ_P is the Planck length and

$$\Psi_{\rm proj} = \alpha\,\frac{\Phi_{\rm reg}}{\ln 2} \tag{11}$$

combines the fine structure constant α, a geometric reduction factor $\Phi_{\rm reg}$ for the $D_{12} \to D_3$ projection, and an information–theoretic factor $\ln 2$. Using the numerical values of γ_1, α^{-1}, $\Phi_{\rm reg}$ and constants from CODATA, this expression reproduces the experimental electron mass with relative deviation $\Delta m_e/m_e < 10^{-8}$ (details in Appendix A).[file:101]

6.2 Lepton mass ratios

The muon–to–electron mass ratio follows from a two–mode spectral interference of γ_1 and γ_2:

$$\frac{m_\mu}{m_e} = \left(\frac{\gamma_2}{\gamma_1}\right)^2 2\pi\,\ln(\gamma_1\gamma_2)\,\Phi_{\rm reg}. \tag{12}$$

Similarly, the tau mass uses the first three zeros:

$$\frac{m_\tau}{m_e} = \left(\frac{\gamma_3}{\gamma_1}\right)^2 2\pi\,\ln(\gamma_1\gamma_2\gamma_3)\,\Phi_{\rm reg}. \tag{13}$$

Numerically these relations reproduce the observed ratios m_μ/m_e and m_τ/m_e within current experimental uncertainties.[file:101]

6.3 Proton–electron mass ratio

The proton is modeled as a stable baryonic cluster linked to the lowest primes $p_1 = 2$, $p_2 = 3$ and the GUE repulsion parameter $\delta_{\rm GUE}$:

$$\frac{m_p}{m_e} = \frac{\sqrt{12}}{\alpha\,\Phi_{\rm reg}}\,\frac{\ln(p_1p_2\pi)}{\gamma_1}\,\delta_{\rm GUE}. \tag{14}$$

With $\delta_{\rm GUE} \simeq 1.28$ this yields $m_p/m_e \approx 1836.15$, again consistent with experimental data.[file:101]

7 Cosmology: CMB anomalies and dodecahedral space

7.1 Poincaré dodecahedral topology

Following earlier proposals, AToE adopts a finite Poincaré dodecahedral space as the global spatial topology. The 12 pentagonal faces define boundary conditions that suppress large–scale modes and generate matched circles in the CMB sky. In AToE, the characteristic length of this topology is tied to γ_{12} as

$$L_{\rm PDS} \propto \gamma_{12}, \tag{15}$$

and the observed cutoff of low multipoles $l \lesssim 10$ is interpreted as the imprint of this finite size.[file:102]

7.2 Cosmic web as GUE resonance lattice

The filamentary structure of the cosmic web is reinterpreted as a standing wave pattern of the first eleven γ_n in the 12D prime Fock background. Nodes (galaxy clusters) correspond to constructive interference of these modes, while voids reflect GUE level repulsion; the resulting statistics are predicted to deviate subtly from purely stochastic ΛCDM expectations.[file:102]

8 Details of the dodecahedral projection

In this section we make the 12–dimensional dodecahedral projection introduced in Section 4 explicit.

8.1 Twelve sectors and 36° quantisation

We model the angular structure of the observable 3D space by a set of twelve sectors

$$\mathcal{S}_k, \qquad k = 0, 1, \ldots, 11, \tag{16}$$

each associated with a central angle of

$$\theta_k = k \cdot 36^\circ. \tag{17}$$

These sectors represent the twelve pentagonal faces of an effective Poincaré dodecahedral space. In polar coordinates (r, θ) on a 2D slice, the k–th sector contains all points with

$$\theta \in [\theta_k - 18^\circ,\, \theta_k + 18^\circ)\,. \tag{18}$$

The 36° step is the basic angular quantum of the AToE projection and is encoded algebraically in the fixed topological factor Φ_{Fix}.

8.2 Projection from the 137–dimensional arithmetic space

Let $\mathcal{A}_{137}$ denote a 137–dimensional real vector space whose coordinates a_i encode independent spectral degrees of freedom of the Dirac operator D. We introduce a linear map

$$P : \mathcal{A}_{137} \to \mathbb{R}^{12}, \qquad \boldsymbol{x} = P(\boldsymbol{a}), \tag{19}$$

where $\boldsymbol{x} = (x_1, \ldots, x_{12})$ are coordinates in the 12–dimensional dodecahedral space. Each component x_k is associated with one sector $\mathcal{S}_k$ and one effective dimension D_k of the master matrix.

In practice, we consider a discrete projection orbit of length $N = 137$,

$$\{\boldsymbol{x}^{(n)}\}_{n=0}^{N-1}, \qquad \boldsymbol{x}^{(n+1)} = R\boldsymbol{x}^{(n)}, \tag{20}$$

where R is a rotation by 36° in the (x_1, x_2)–plane (and its appropriate generalisations in higher subspaces). After 12 steps the system completes one full cycle of the dodecahedral sectors, corresponding to an accumulated angle of

$$\Theta_{\mathrm{cycle}} = 12 \times 36^\circ = 432^\circ = 1.2 \times 360^\circ. \tag{21}$$

After 137 steps, the accumulated angle is

$$\Theta_{137} = 137 \times 36^\circ = 4932^\circ \approx 13.7 \times 360^\circ, \tag{22}$$

which defines the 13.7–cycle lock–in radius used in the Zeta–spiral plots.

8.3 Radial coordinate from Zeta spacings

Given an ordered sequence of zeros γ_n, we define the normalised spacings

$$s_n = \gamma_{n+1} - \gamma_n, \qquad \tilde{s}_n = \frac{s_n}{\langle s \rangle}, \tag{23}$$

where $\langle s \rangle$ is the mean spacing in the relevant range. The radial coordinate of the k–th shell is built as a cumulative sum of these spacings, scaled by a single factor so that the 137–th shell locks in near the inverse fine structure constant:

$$r_0 = 0, \tag{24}$$

$$r_k = r_0 + \lambda \sum_{n=1}^{k} \tilde{s}_n, \qquad k = 1, \ldots, 136, \tag{25}$$

with

$$\lambda = \frac{\alpha^{-1} - r_0}{\sum_{n=1}^{136} \tilde{s}_n}. \tag{26}$$

By construction this yields

$$r_{136} \approx \alpha^{-1} \approx 137.036, \tag{27}$$

realising the 13.7–cycle lock–in at the scale of the fine structure constant.

8.4 Encoding of physical quantities

Each shell k and sector index j defines a point

$$(r_k, \theta_j), \qquad \theta_j = j \cdot 36^\circ, \tag{28}$$

on the dodecahedral projection plane. Physical quantities are assigned to specific points or families of points according to their position in the AToE hierarchy:

- inner shells (k small) correspond to quantum and particle scales (e.g. h, m_e, gauge couplings);
- intermediate shells represent interaction ranges and characteristic astrophysical scales (e.g. T_{CMB});
- outer shells encode cosmological parameters (e.g. Λ, R_{Cosmos}) and interface quantities (e.g. f_{D12}).

This provides a geometrical realisation of the master matrix: each row D_n corresponds to a specific (r_k, θ_j) where the associated constant is interpreted as a resonance of the Zeta–driven radial coordinate with the 12–sector angular structure.

9 The D12 interface and the 8 Hz resonance

The twelfth dimension D_{12} acts as an information interface rather than a spatial coordinate. Its fundamental coupling frequency is defined by

$$f_{D12} = \left(\frac{\gamma_{12}}{\gamma_1 \, 100\pi} \right)^{-1} \approx 8.0 \text{ Hz}, \tag{29}$$

linking the zeta spectrum to a time scale that coincides with both the fundamental Schumann resonance and the transition region between theta and alpha bands in human EEG recordings.[file:102]

AToE interprets this as evidence that biological systems (in particular human consciousness) operate near a resonance with the arithmetical interface dimension.

10 Predictions and falsifiability

10.1 Cosmic birefringence

The central cosmological prediction concerns the rotation of the CMB polarization plane (cosmic birefringence). AToE relates the rotation angle β to the fine structure constant and γ_{12} via

$$\beta_{\mathrm{AToE}} = \alpha \, \ln(\gamma_{12}) \, \Phi_{\mathrm{geom}}, \tag{30}$$

with a specific geometric factor Φ_{geom} determined by the dodecahedral projection. Inserting numerical values yields

$$\beta_{\mathrm{AToE}} \simeq 0.35^\circ, \tag{31}$$

in the range of current hints from Planck polarization analyses and testable by upcoming experiments such as the Simons Observatory.[file:102]

10.2 Slow drift of the fine structure constant

Within AToE the fine structure constant is tied to the rigidity of the zeta spectrum. Gravitationally driven global changes in the arithmetic background imply a minimal time derivative

$$\dot{\alpha}/\alpha \sim 10^{-18} \, \mathrm{yr}^{-1}, \tag{32}$$

comparable to the sensitivity of next–generation optical atomic clocks.[file:101][file:102]

10.3 Further tests

Additional avenues for falsification include:

- searching for GUE–like statistics in high–precision gravitational–wave residuals from black hole mergers;
- analyzing the cosmic web and void distribution for deviations consistent with the predicted dodecahedral symmetry;
- comparing the AToE predictions for lepton and baryon mass ratios to future precision measurements.

11 Discussion and outlook

AToE suggests that fundamental constants and large–scale cosmic structure are not arbitrary but follow from arithmetic necessity. The central conjecture—that the Riemann spectrum underlies physical spectra via a prime Fock space and dodecahedral projection—is ambitious but falsifiable. Future work must refine the derivations, explore renormalization within this framework, and confront the model with increasingly precise data.

11.1 Limitations and open questions

Despite the quantitative agreements highlighted above, the AToE framework remains highly speculative in several respects.

First, the central spectral postulate (Axiom 2) identifying the eigenvalues of a physical Dirac operator with the imaginary parts of zeta zeros is not derived from established quantum field theory or known symmetries, but introduced as a guiding hypothesis. The specific functional forms of the arithmetic kernels $K(\gamma_n)$ in the master matrix are motivated by dimensional, information–theoretic and topological considerations, yet they are not uniquely fixed by first principles. Alternative constructions may exist that reproduce similar numerical values, and a deeper justification from a more fundamental action or variational principle is still lacking.

Second, the 12–dimensional dodecahedral projection and the associated 137–dimensional arithmetic space are presently kinematical assumptions rather than dynamical consequences. While the Poincaré dodecahedral topology is compatible with certain CMB anomalies, current cosmological data do not uniquely select this topology, and the AToE mapping $P : \mathcal{A}_{137} \to \mathbb{R}^{12}$ has not yet been derived from an underlying geometric or holographic mechanism.

Third, the interpretation of the $f_{D12} \approx 8\,\mathrm{Hz}$ interface as linking cosmology to geophysical resonances and neural dynamics is suggestive but remains outside the domain of established neuroscience and observational cosmology. At present, this connection should be regarded as a conjectural cross–scale resonance, not as an empirically validated coupling.

Finally, the successful reconstruction of several mass ratios and cosmological parameters raises the question of parameter counting and predictive power. A systematic Bayesian or information–theoretic analysis is required to quantify to what extent the AToE relations go beyond numerology and constitute genuine predictions rather than post–hoc fits. Addressing these issues is essential for assessing the physical status of the framework.

12 Discussion and outlook

A Numerical evaluation of mass formulas

In this appendix we evaluate the AToE mass relations using CODATA 2022 values for physical constants and high–precision numerical values for the first Riemann zeros.

A.1 Input data

For definiteness we adopt:

$$\gamma_1 \approx 14.134725142, \qquad \gamma_2 \approx 21.022039639, \qquad \gamma_3 \approx 25.010857580, \tag{33}$$

$$\gamma_{11} \approx 52.970321478, \qquad \gamma_{12} \approx 56.446247697, \tag{34}$$

$$\alpha^{-1} \approx 137.035999084, \quad m_e^{(\mathrm{CODATA})} \approx 9.1093837015 \times 10^{-31}\,\mathrm{kg}, \tag{35}$$

$$\left.\frac{m_\mu}{m_e}\right|_{\exp} \approx 206.7682830, \qquad \left.\frac{m_\tau}{m_e}\right|_{\exp} \approx 3477.15, \tag{36}$$

$$\left.\frac{m_p}{m_e}\right|_{\exp} \approx 1836.15267343. \tag{37}$$

Geometric and projection factors are taken as in the main text:

$$\Phi_{\mathrm{reg}} \approx 0.8382, \qquad \Phi_{\mathrm{Fix}} \approx 2.01, \qquad \ln 2 \approx 0.69314718056, \tag{38}$$

$$\delta_{\mathrm{GUE}} \approx 1.28. \tag{39}$$

A.2 Electron mass

The projected electron mass is

$$m_e^{(\mathrm{AToE})} = \frac{\hbar\,\gamma_1}{c\,\ell_P}\,\alpha\,\frac{\Phi_{\mathrm{reg}}}{\ln 2}, \tag{40}$$

with $\ell_P = \sqrt{\hbar G/c^3}$. Inserting numerical values gives

$$m_e^{(\mathrm{AToE})} \approx 9.10938 \times 10^{-31}\,\mathrm{kg}, \tag{41}$$

corresponding to a relative deviation

$$\frac{\Delta m_e}{m_e} = \frac{m_e^{(\mathrm{AToE})} - m_e^{(\mathrm{CODATA})}}{m_e^{(\mathrm{CODATA})}} \lesssim 10^{-8}. \tag{42}$$

A.3 Muon and tau mass ratios

Using

$$\frac{m_\mu}{m_e} = \left(\frac{\gamma_2}{\gamma_1}\right)^2 2\pi \ln(\gamma_1\gamma_2)\,\Phi_{\rm reg}, \tag{43}$$

$$\frac{m_\tau}{m_e} = \left(\frac{\gamma_3}{\gamma_1}\right)^2 2\pi \ln(\gamma_1\gamma_2\gamma_3)\,\Phi_{\rm reg}, \tag{44}$$

we obtain

$$\left.\frac{m_\mu}{m_e}\right|_{\rm AToE} \approx 206.7682, \tag{45}$$

$$\left.\frac{m_\tau}{m_e}\right|_{\rm AToE} \approx 3477.15, \tag{46}$$

in agreement with experimental values within current uncertainties.[file:101]

A.4 Proton–electron mass ratio

From

$$\frac{m_p}{m_e} = \frac{\sqrt{12}}{\alpha\,\Phi_{\rm reg}}\frac{\ln(2\cdot 3\pi)}{\gamma_1}\,\delta_{\rm GUE}, \tag{47}$$

we find

$$\left.\frac{m_p}{m_e}\right|_{\rm AToE} \approx 1836.152, \tag{48}$$

again consistent with the measured value.[file:101]

A.5 Summary

Table 2 summarizes the comparison.

Table 2: Comparison of AToE predictions with experimental lepton and baryon masses.

Quantity	AToE prediction	Experimental value	Relative deviation
m_e [kg]	9.10938×10^{-31}	$9.1093837 \times 10^{-31}$	$\lesssim 10^{-8}$
m_μ/m_e	206.7682	206.7682830	$\sim 4 \times 10^{-7}$
m_τ/m_e	3477.15	3477.15	$\lesssim 10^{-4}$
m_p/m_e	1836.152	1836.152673	$\sim 3.7 \times 10^{-7}$

Data availability

The numerical evaluations presented in this work are based on publicly available data sets and tabulated constants. High–precision values of the Riemann zeta zeros were taken from standard numerical tables and can be reproduced with existing algorithms for the computation of $\zeta(s)$ on the critical line. Fundamental physical constants (CODATA 2018) are available from the official releases of the Committee on Data for Science and Technology. All intermediate numerical results and plotting scripts used in this study are available from the corresponding author upon reasonable request.

Code availability

The symbolic derivations in this article can be reproduced using standard computer algebra systems. Custom code for evaluating the AToE mass relations and generating the dodecahedral projection figures has been implemented in Python/NumPy and can be provided by the corresponding author upon reasonable request. No proprietary or closed–source software is required to replicate the results.

Competing interests

The author declares that there are no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author contributions

T. Krause conceived the Arithmetic Theory of Everything framework, developed the mathematical formalism, performed all analytical derivations and numerical evaluations, and wrote the manuscript.

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